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$$\therefore\ \sec\beta=\frac{e[\sqrt{(2e^2-1)}-1]}{e^2-1}.\quad \therefore\ \frac{e\sqrt{(2)}[\sqrt{(2e^2-1)}-1]}{2(e^2-1)}\text{ is semi-major axis.}$$

$$\therefore\ \text{average volume is }\tfrac{1}{2}e\left(\frac{e\sqrt{(2)}[\sqrt{(2e^2-1)}-1]}{2(e^2-1)}+\tfrac{1}{2}\right)\tfrac{1}{2}\pi=\triangle.$$

$$\therefore\ \triangle=\frac{\pi e}{8(e^2-1)}[e\sqrt{2}(\sqrt{2e^2-1}-1)+e^2-1]=5.4345\text{ cubic inches, when}$$

$e=5$ inches,

72. Proposed by B. F. FINKEL, A.M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A rod is broken at random into four pieces; find the chance that no one of the pieces is greater than the sum of the other three. [From *C. Smith's Treatise on Algebra*, p. 528.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let a=length of rod.

By conditions of problem no part can be greater than $\frac{1}{2}a$.

Let $ABCD-G$ be a cube side a.

Let $Abcd-g$ be a cube side $\frac{1}{2}a$.

For favorable cases the points are confined to the smaller cube.

$$\therefore\ \text{chance}=\frac{(\frac{1}{2}a)(\frac{1}{2}a)(\frac{1}{2}a)}{(a)(a)(a)}=\tfrac{1}{8}.$$

73. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

On an average 1 vessel out of every n is wrecked. Find the chance that out of m vessels expected p at least will arrive safely.

I. Solution by the PROPOSER.

The chance of a vessel arriving is $[(n-1)/n]$.

The chance of a vessel not arriving is $1/n$.

The event will happen if, m, $(m-1)$, $(m-2)$, $(m-3)$, $(m-4)$, ... down to p vessels arrive.

Thus the required chance is the sum of the first $(m-p+1)$ terms in the expansion of

$$\left(\frac{n-1}{n}+\frac{1}{n}\right)^m=\left(\frac{n-1}{n}\right)^m+\frac{m}{1}\left(\frac{1}{n}\right)\left(\frac{n-1}{n}\right)^{m-1}+\frac{m(m-1)}{2!}\left(\frac{1}{n}\right)^2\left(\frac{n-1}{n}\right)^{m-2}$$

$$+\ \cdots\cdots\frac{m!}{p!(m-p)!}\left(\frac{1}{n}\right)^{m-p}\left(\frac{n-1}{n}\right)^p.$$

If $n=10$, $m=5$, $p=3$, we get chance$=(\frac{9}{10})^5+5(\frac{1}{10})(\frac{9}{10})^4+10(\frac{1}{10})^2(\frac{9}{10})^3$ $=\frac{12393}{12500}$.

II. Solution by HENRY HEATON, M. Sc., Atlantic, Ia.

For convenient notation put $n-1=k$.

Then the probability that any given vessel will be wrecked is $1/n$, that it will be saved, k/n.

The probability that all the m vessels will be saved is k^m/n^m.

The probability that any particular vessel will be lost while the others are saved is $(k/n)^{m-1}(1/n)$.

There are m vessels. Hence the probability that *one* vessel will be lost while the others are saved is $m(k^{m-1}/n^m)$.

The probability that any two particular vessels will be lost while the others are saved is $(k/n)^{m-2}(1/n)^2$.

The number of different combinations for two vessels to be saved and the rest lost is $\frac{m(m-1)}{2!}$.

Hence the probability that two vessels will be lost is $\frac{m(m-1)}{2!}\frac{k^{m-2}}{n^m}$.

In like manner the probability that 3 vessels will be lost and $m-3$ saved is $\frac{m(m-1)(m-2)k^{m-3}}{n^m 3!}$ and that 1, 2, 3, ... or $m-p$ vessels will be lost is

$$P=\frac{1}{n^m}\left(k^m+mk^{m-1}+\frac{m(m-1)k^{m-2}}{2!}\ldots\ldots+\frac{m(m-1)\ldots\ldots(p-1)k^p}{(m-p)!}\right.$$

If $p=m$, $P=\left(\frac{k}{n}\right)^m$. If $p=0$, $P=\frac{(k+1)^m}{n^m}=1$, as it evidently should.

III. Solution by P. H. PHILBRICK, C. E., Lake Charles, La.

The chance that any particular ship arrives is $\frac{n-1}{n}$. The chance that any particular set of p ships arrives is $\left(\frac{n-1}{n}\right)^p$; and the chance that the other $m-p$ ships fails is $\left(\frac{1}{n}\right)^{m-p}$. Hence the chance that any particular set of p ships alone arrives is $\left(\frac{n-1}{n}\right)^p\left(\frac{1}{n}\right)^{m-p}$. But out of m ships p can be selected in $\frac{m(m-1)\ldots\ldots(m-p+1)}{1.2\ldots\ldots p}$ ways. Hence the chance that some possible set of p ships will arrive is

$$\left(\frac{n-1}{n}\right)^p\cdot\left(\frac{1}{n}\right)^{m-p}\cdot\frac{m(m-1)\ldots\ldots(m-p+1)}{1.2\ldots\ldots p}\ldots\ldots(1).$$

Writing $p+1$, $p+2$, ...m, successively, in place of p in (1) will give the chances that $p+1$, $p+2$, ... m ships will arrive. The sum of these added to (1) gives the chance that *at least* p ships out of m will arrive.

Also solved by *ELMER SCHUYLER*.

MISCELLANEOUS.

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71. **Proposed by GUY B. COLLIER, 1901 Union, 27 Middle Section of South College, Schenectady, N. Y.**

Find the locus of any point on the front sprocket of a bicycle during one revolution of the hind wheel (any gear may be assumed).

I. **Solution by R. E. GAINES, A. M., Professor of Mathematics, Richmond, Va.**

Suppose the front sprocket revolves e times as fast as the hind wheel of the bicycle (if, to fix the idea, the gear be 70, then $e=\frac{2}{5}$), and take as axis of x the line on which the bicycle rolls, and as axis of y a line perpendicular to this, and passing through P when it was in its lowest position. Then let the hind wheel turn through an angle φ (the sprocket turning through an angle $e\varphi$) and the *center* of the sprocket has moved forward a distance $a\varphi$ and we obtain at once

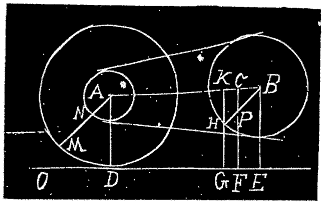
$$x=a\varphi-r\sin(e\varphi),\quad y=a-r\cos(e\varphi),$$

where r=radius of sprocket, a=radius of hind wheel, and φ=angle through which the hind wheel has turned. Expressed as a single equation, this is

$$x=(a/e)\cos^{-1}\frac{a-y}{r}-r\sqrt{1-\left(\frac{a-y}{r}\right)^2}.$$

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

Let O be the origin, $OF=x$, $PF=y$, P=any point in the front sprocket, g=gear, $AD=a$, radius of hind wheel, $AB=b$, $BP=c$, and also let $BE=AD=a$, $\angle MAD=\theta$.



Then $BH/AN=g/2a$.

$\therefore\ \angle HBE=(2a/g)\theta$.

$\therefore\ x=OD+DE-FE=OD+AB-BC$.

$\therefore\ x=b+a\theta-c\sin(2a\theta/g)$.

$\therefore\ y=a-c\cos(2a\theta/g)$.

$\therefore\ x=b+(\frac{1}{2}g)\cos^{-1}\left(\frac{a-y}{c}\right)-\sqrt{c^2-(a-y)^2}$ is the equation to the locus.

s=length for one revolution of hind wheel.

$$\therefore\ s=(1/g)\int_0^{2\pi}\sqrt{a^2g^2+4a^2c^2-4a^2cg\cos(2a\theta/g)}\,d\theta$$